1. <u>Attendance Problem:</u> Find the midpoint and slope of a segment with endpoints (2, 8) & (-4, 6).

- I can prove and apply theorems about perpendicular bisectors.
- I can prove and apply theorems about angle bisectors.

Common Core

CC.9-12.G.CO.9 Prove geometric theorems about lines and angles.
CC.9-12.G.SRT.4 Prove theorems about triangles.

Vocabulary	
equidistant	locus

Sketchpad Activities

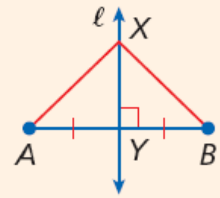
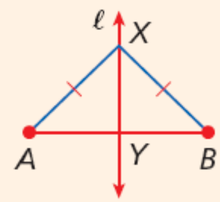
- Perpendicular bisector
- Angle bisector

When a point is the same distance from two or more objects, the point is said to be **<u>equidistant</u>** from the objects. Triangle congruence theorems can be used to prove theorems about equidistant points.

Q: Where do math teachers shop?
A: Deci-malls (decimals)!

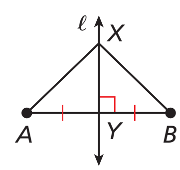
"I go on working for the same reason a hen goes on laying eggs." - *H. L. Mencken*

Theorems: Distance and Perpendicular Bisectors

	THEOREM	HYPOTHESIS	CONCLUSION
5-1-1	**Perpendicular Bisector Theorem** If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.	 $\overline{XY} \perp \overline{AB}$ $\overline{YA} \cong \overline{YB}$	$XA = XB$
5-1-2	**Converse of the Perpendicular Bisector Theorem** If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.	 $XA = XB$	$\overline{XY} \perp \overline{AB}$ $\overline{YA} \cong \overline{YB}$

A **locus** is a set of points that satisfies a given condition. The perpendicular bisector of a segment can be defined as the locus of points in a plane that are equidistant from the endpoints of the segment.

Perpendicular Bisector Theorem



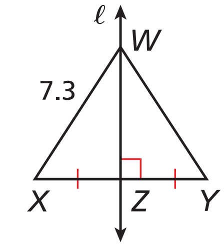
Given: ℓ is the perpendicular bisector of $\overline{AB}$.
Prove: $XA = XB$

Proof:

Since ℓ is the perpendicular bisector of $\overline{AB}$, $\ell \perp \overline{AB}$ and Y is the midpoint of $\overline{AB}$. By the definition of perpendicular, $\angle AYX$ and $\angle BYX$ are right angles and $\angle AYX \cong \angle BYX$. By the definition of midpoint, $\overline{AY} \cong \overline{BY}$. By the Reflexive Property of Congruence, $\overline{XY} \cong \overline{XY}$. So $\triangle AYX \cong \triangle BYX$ by SAS, and $\overline{XA} \cong \overline{XB}$ by CPCTC. Therefore $XA = XB$ by the definition of congruent segments.

1 Applying the Perpendicular Bisector Theorem and Its Converse

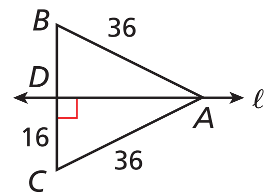
Find each measure.



A YW

$YW = XW$ — *⊥ Bisector Thm.*

$YW = \mathbf{7.3}$ — *Substitute 7.3 for XW.*

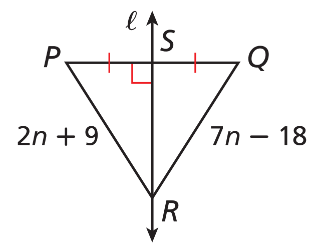


B BC

Since $AB = AC$ and $\ell \perp \overline{BC}$, ℓ is the perpendicular bisector of $\overline{BC}$ by the Converse of the Perpendicular Bisector Theorem.

$BC = 2CD$ — *Def. of seg. bisector*

$BC = 2(\mathbf{16}) = 32$ — *Substitute 16 for CD.*



C PR

$PR = RQ$ — *⊥ Bisector Thm.*

$\mathbf{2n + 9 = 7n - 18}$ — *Substitute the given values.*

$9 = 5n - 18$ — *Subtract 2n from both sides.*

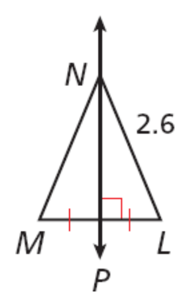
$27 = 5n$ — *Add 18 to both sides.*

$5.4 = n$ — *Divide both sides by 5.*

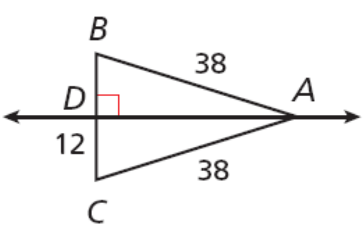
So $PR = 2(\mathbf{5.4}) + 9 = 19.8$.

<u>Example 1</u>. Find each measure.

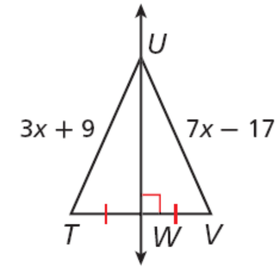
A. MN



B. BC

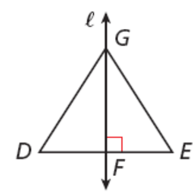


C. TU



<u>Guided Practice</u>. Find the measure.

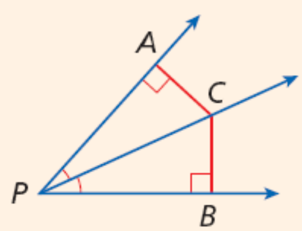
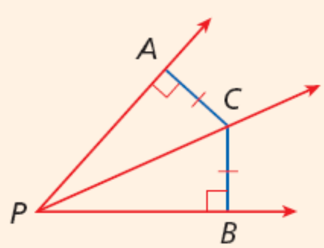
14. Given that line ℓ is the perpendicular bisector of $\overline{DE}$ and $EG = 14.6$, find DG.



15. Given that $DE = 20.8$, $DG = 36.4$, and $EG = 36.4$, find EF.

Remember that the distance between a point and a line is the length of the perpendicular segment from the point to the line.

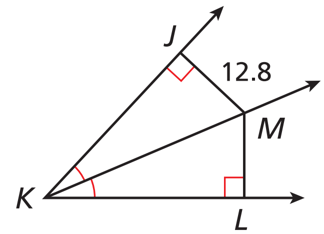
Theorems Distance and Angle Bisectors

THEOREM	HYPOTHESIS	CONCLUSION
5-1-3 **Angle Bisector Theorem** If a point is on the bisector of an angle, then it is equidistant from the sides of the angle.	 $\angle APC \cong \angle BPC$	$AC = BC$
5-1-4 **Converse of the Angle Bisector Theorem** If a point in the interior of an angle is equidistant from the sides of the angle, then it is on the bisector of the angle.	 $AC = BC$	$\angle APC \cong \angle BPC$

Based on these theorems, an angle bisector can be defined as the locus of all points in the interior of the angle that are equidistant from the sides of the angle.

2 Applying the Angle Bisector Theorems

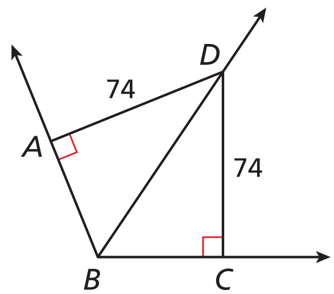
Find each measure.



A LM

$LM = JM$ — *∠ Bisector Thm.*

$LM = 12.8$ — *Substitute 12.8 for JM.*

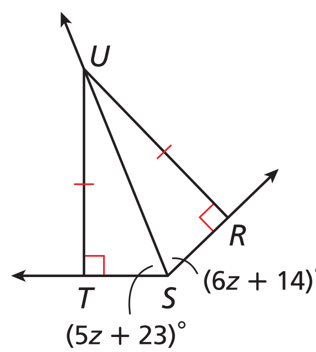


B **m∠*ABD*, given that m∠*ABC* = 112°**

Since $AD = DC$, $\overline{AD} \perp \overline{BA}$, and $\overline{DC} \perp \overline{BC}$, $\overrightarrow{BD}$ bisects $\angle ABC$ by the Converse of the Angle Bisector Theorem.

$m\angle ABD = \frac{1}{2} m\angle ABC$ — *Def. of ∠ bisector*

$m\angle ABD = \frac{1}{2}(112°) = 56°$ — *Substitute 112° for m∠ABC.*



C **m∠*TSU***

Since $RU = UT$, $\overline{RU} \perp \overline{SR}$, and $\overline{UT} \perp \overline{ST}$, $\overrightarrow{SU}$ bisects $\angle RST$ by the Converse of the Angle Bisector Theorem.

$m\angle RSU = m\angle TSU$ — *Def. of ∠ bisector*

$6z + 14 = 5z + 23$ — *Substitute the given values.*

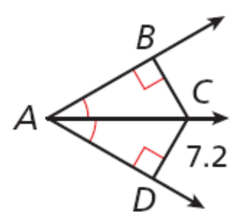
$z + 14 = 23$ — *Subtract 5z from both sides.*

$z = 9$ — *Subtract 14 from both sides.*

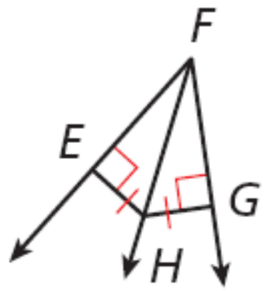
So $m\angle TSU = [5(9) + 23]° = 68°$.

<u>Example 2</u>. Find each measure.

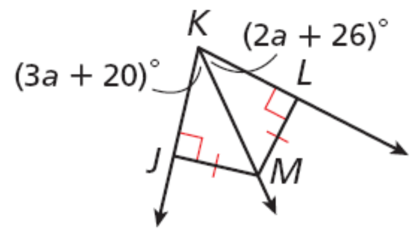
A. BC



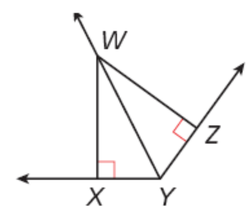
B. $m\measuredangle EFH$ given that $m\measuredangle EFG = 50°$



C. $m\measuredangle MKL$



Guided Practice. Find the measure.

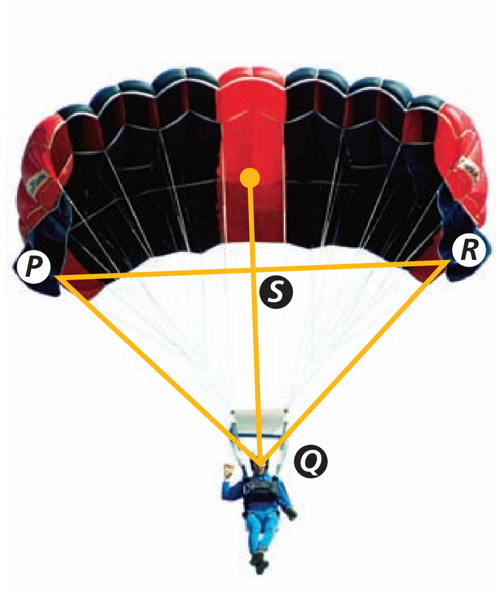


16. Given that $\overrightarrow{YW}$ bisects $\measuredangle XYZ$ and $WZ = 3.05$, find WX.

17. Given that $m\measuredangle WYZ = 63°$, , $XW = 5.7$, and $ZW = 5.7$, find $m\measuredangle XYZ$..

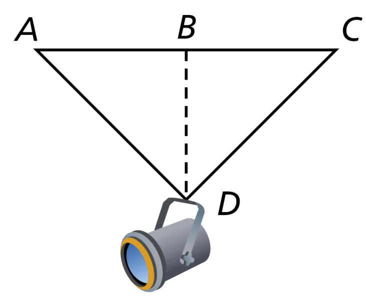
3 *Parachute Application*

Each pair of suspension lines on a parachute are the same length and are equally spaced from the center of the chute. How do these lines keep the sky diver centered under the parachute?

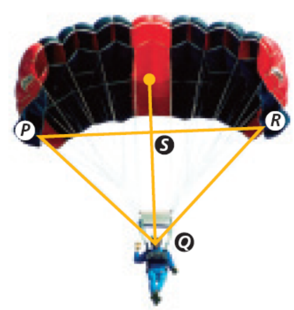


It is given that $\overline{PQ} \cong \overline{RQ}$. So Q is on the perpendicular bisector of $\overline{PR}$ by the Converse of the Perpendicular Bisector Theorem. Since S is the midpoint of $\overline{PR}$, $\overline{QS}$ is the perpendicular bisector of $\overline{PR}$. Therefore the sky diver remains centered under the chute.

Example 3. John wants to hang a spotlight along the back of a display case. Wires $\overline{AD}$ and $\overline{CD}$ are the same length, and A and C are equidistant from B. How do the wires keep the spotlight centered?

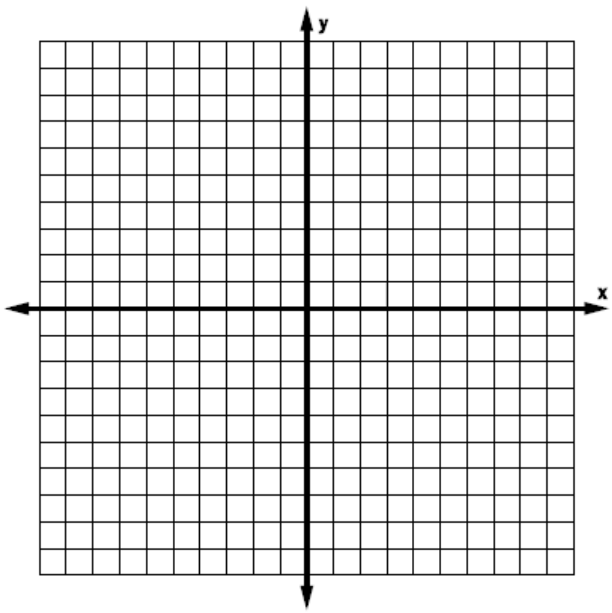


18. Guided Practice. S is equidistant from each pair of suspension lines. What can you conclude about $\overrightarrow{QS}$?



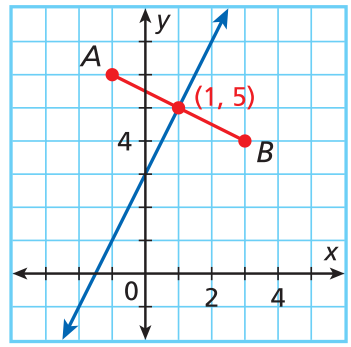
Refer to video example 4. (my.hrw.com)

Write an equation in point-slope form for the perpendicular bisector of the segment with endpoints *A*(−1, 4) and *B*(3, 6).



4 Writing Equations of Bisectors in the Coordinate Plane

Write an equation in point-slope form for the perpendicular bisector of the segment with endpoints $A(-1, 6)$ and $B(3, 4)$.



Step 1 Graph $\overline{AB}$.

The perpendicular bisector of $\overline{AB}$ is perpendicular to $\overline{AB}$ at its midpoint.

Step 2 Find the midpoint of $\overline{AB}$.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$ *Midpoint formula*

$$\text{mdpt. of } \overline{AB} = \left(\frac{-1 + 3}{2}, \frac{6 + 4}{2}\right) = (1, 5)$$

Step 3 Find the slope of the perpendicular bisector.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$ *Slope formula*

$$\text{slope of } \overline{AB} = \frac{4 - 6}{3 - (-1)} = \frac{-2}{4} = -\frac{1}{2}$$

Since the slopes of perpendicular lines are opposite reciprocals, the slope of the perpendicular bisector is 2.

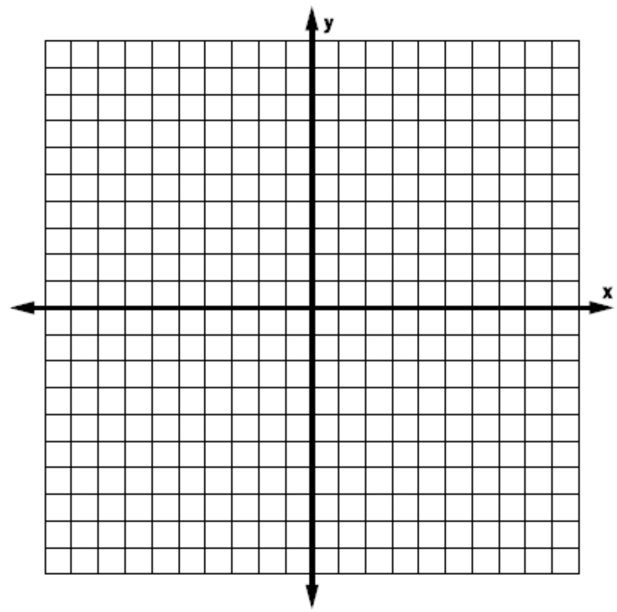
Step 4 Use point-slope form to write an equation.

The perpendicular bisector of $\overline{AB}$ has slope 2 and passes through $(1, 5)$.

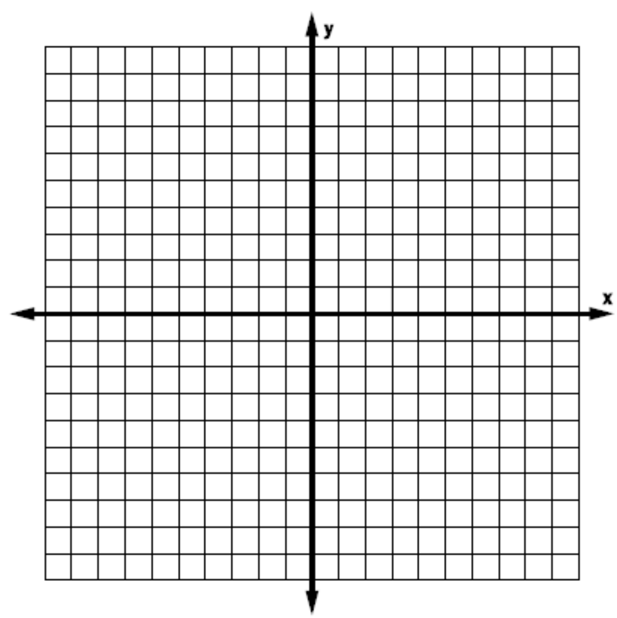
$$y - y_1 = m(x - x_1)$$ *Point-slope form*

$$y - 5 = 2(x - 1)$$ *Substitute 5 for y_1, 2 for m, and 1 for x_1.*

<u>Example 4</u>. Write an equation in point-slope form for the perpendicular bisector of the segment with endpoints *C*(6, –5) and *D*(10, 1).



19. <u>Guided Practice</u>. Write an equation in point-slope form for the perpendicular bisector of the segment with endpoints *P*(5, 2) and *Q*(1, –4).



5-1 Perpendicular and angle bisectors *(p 316)* 13, 15, 17, 18, 20, 21, 24, 26.